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Cosmological Particle Creation and Dynamical Casimir Effect

M. R. Setare

We compute particle creation for a real massive scalar field conformally coupled to a spatially closed Robertson–Walker space-time background, with time-dependent scale factor. This is a dynamical Casimir effect with moving boundaries.

KEY WORDS: dynamical Casimir effect; particle creation; scaler field; vacuum.

1. INTRODUCTION

The Casimir effect is one of the most interesting manifestations of nontrivial properties of the vacuum state in a quantum field theory (for reviews see Birrell and Davies, 1982; Milton, 1999; Mostepanenko and Trunov, 1997; Plunein *et al.*, 1986) and can be viewed as a polarization of vacuum by boundary conditions. A new phenomenon, a quantum creation of particles (the dynamical Casimir effect) occurs when the geometry of the system varies in time. In two-dimensional spacetime and for conformally invariant fields the problem with dynamical boundaries can be mapped to the corresponding static problem and hence allows a complete study (see Birrell and Davies, 1982; Mostepanenko and Trunov, 1997 and references therein). In higher dimensions the problem is much more complicated and is solved for some simple geometries. The vacuum stress induced by uniform acceleration of a perfectly reflecting plane is considered in (Candelas and Deutsch, 1977). The corresponding problem for a sphere expanding in the four-dimensional spacetime with constant acceleration is investigated by Frolov and Serebriany (Frolov and Serebriany, 1979, 1980) in the perfectly reflecting case and by Frolov and Singh (Frolov and Singh, 1999) for semi-transparent boundaries. For more general cases of motion by vibrating cavities the problem of particle and energy creation is considered on the base of various perturbation methods (Calucci, 1992; Dodonov and Klimov, 1996; Dodonov, 1998; Jauregui *et al.*, 1995; Ji *et al.*, 1997; Lambrecht *et al.*, 1996; Sassaroli *et al.*, 1994; Schutzhold *et al.*, 1998) (for more complete list

[1] Physics Department, Institute for Studies in Theoretical Physics and Mathematics (IPM), P. O. Box 19395-5531, Tehran; e-mail: rezakord@ipm.ir.

of references see Dodonov, 1998). It have been shown that a gradual accumulation of small changes in the quantum state of the field could result in a significant observable effect. A new application of the dynamical Casimir effect has recently appeared in connection with the suggestion by Schwinger (Schwinger, 1993, 1994) that the photon production associated with changes in the quantum electrodynamic vacuum state arising from a collapsing dielectric bubble could be relevant for sonoluminescence (the phenomenon of light emission by a sound-driven gas bubble in a fluid Barber *et al.*, 1997). For further developments and discussions on this quantum–vacuum approach see Eberlein (1996), Liberati *et al.* (2000a,b), Milton (1995, 1998) and references therein.

The possibility of particle production due to space-time curvature has been discussed by Schrodinger (Schrodinger, 1939), while other early work is due to DeWitt (DeWitt, 1953), and Imamura (Imamura, 1960). The first thorough treatment of particle production by an external gravitational field was given by Parker (Parker, 1968, 1969). Particle creation from the quantum scalar vacuum by expanding or contracting spherical shell with Dirichlet boundary conditions is considered in (Setare and Saharian, 2001). In another paper the case is considered when the sphere radius performs oscillation with a small amplitude and the expression is derived for the number of created particles to the first order of the perturbation theory (Setare and Saharian, 2001). Now in the present paper by using the result of (Setare and Saharian, 2001) we consider particle creation in closed Robertson–Walker space-time, when the scale factor represent an asymptotically static space-time.

2. GRAVITATIONAL PARTICLE CREATION

In flat space-time, Lorentz invariance is a guide which generally allows to identify a unique vacuum state for the theory. However, in curved space-time, we do not have Lorentz symmetry. In general, there does not exist a unique vacuum state in a curved space-time. As a result, the concept of particles becomes ambiguous, and the problem of the physical interpretation becomes much more difficult (Ford, 1997; Milton, 1995). The particle creation by an expanding universe was first hinted in the work of Schrodinger (Schrodinger, 1939), this phenomenon was first carefully investigated by Parker (Parker, 1969, 1971). We restrict our attention to the case of spatially closed Robertson–Walker universe whose metric is as following

$$ds^2 = a^2(\eta)(d\eta^2 - dl^2), \tag{1}$$

$$dl^2 = d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\varphi^2). \tag{2}$$

where $a(\eta)$ is the scale factor and η is conformal time, $0 \le \chi \le \pi$. Let us consider a real massive scalar field which coupled to the closed Robertson–Walker background. With the dependence of the radius of curvature $a(\eta)$ on time, the case under consideration is a dynamical Casimir effect with moving boundaries (Setare and Saharian, 2001). The corresponding wave equation is

$$(\Box + m^2 + \xi R)\phi = 0, \tag{3}$$

where R is the scalar curvature

$$R = \frac{6(a'' + a)}{a^3}, \tag{4}$$

where prime stands for the conformal time-derivative, ξ is a coupling constant, here we consider the conformal coupling $\xi = 1/6$, in this case the (3) as (Bordag *et al.*, 2001)

$$\phi''(x) + \frac{2a'}{a}\phi'(x) - \Delta^{(3)}\phi(x) + \left(m^2a^2 + \frac{a''}{a} + 1\right)\phi(x) = 0, \tag{5}$$

where $\Delta^{(3)}$ is the angular part of the Laplacian operator on a three-sphere. The solutions of (5) are

$$\phi^{(+)}_{\lambda lM}(x) = \frac{1}{\sqrt{2}a(\eta)} g_\lambda(\eta)\phi\star_{\lambda lM}(\chi, \theta, \varphi). \tag{6}$$

The eigenfunctions of the three-dimensional Laplacian are as

$$\phi_{\lambda lM}(\chi, \theta, \varphi) = \frac{1}{\sqrt{\sin\chi}}\sqrt{\frac{\lambda(\lambda + l)!}{(\lambda - l + 1)!}}P^{-l-1/2}_{\lambda-1/2}(\cos\chi)Y_{lM}(\theta, \varphi), \tag{7}$$

$\lambda = 1, 2, \ldots, L = 0, 1, 2, \ldots, \lambda - 1$,$Y_{l,M}$ are spherical harmonics, and $P^\nu_\mu(z)$ are the adjoint Legendre functions on the cut. The time-dependent function g_λ satisfies the oscillatory equation (Bordag *et al.*, 2001)

$$g''_\lambda(\eta) + \omega^2_\lambda(\eta)g_\lambda(\eta) = 0, \tag{8}$$

where

$$\omega^2_\lambda(\eta) = \lambda^2 + m^2a^2(\eta). \tag{9}$$

Let us consider an exactly solvable case when

$$a(\eta) = \sqrt{A + B\tanh\frac{\eta}{\eta_0}} \quad A > B, \tag{10}$$

where A, B and η_0 are constants, this corresponds to the contraction for $B < 0$ and expansion for $B > 0$. The corresponding frequencies are

$$\omega^2_\lambda(\eta) = \lambda^2 + m^2\left(A + B\tanh\frac{\eta}{\eta_0}\right). \tag{11}$$

For asymptotically static situation at past and future the in- and out- vacuum states can be defined, where we use the notations

$$\omega_\lambda^{\text{in}} = \sqrt{\lambda^2 + m^2 a_-}, \qquad \omega_\lambda^{\text{out}} = \sqrt{\lambda^2 + m^2 a_+}, \qquad a_\pm = \lim_{\eta \to \pm\infty} a(\eta) \tag{12}$$

for the corresponding eigenfrequencies. Now we need to solve the equation (8) with $\omega_\lambda(\eta)$ given by (9). The corresponding solutions are given by hypergeometric function. The normalized in- and out- modes are given by formula (Birrell and Davies, 1982)

$$g_\lambda^s(\eta) = \left(2\omega_\lambda^s\right)^{-1/2} \exp[-\imath\omega_\lambda^+ \eta - \imath\omega_\lambda^- \eta_0 \ln[2\cosh(\eta/\eta_0)]] \times {}_2F_1\big(1 + \imath\omega_\lambda^- \eta_0,$$

$$\times \imath\omega_\lambda^- \eta_0; 1 \mp \imath\omega_\lambda^s \eta_0; \frac{1}{2}(1 \pm \tanh(\eta/\eta_0))\big), \quad s = \text{in, out}, \tag{13}$$

where uper/lower sign corresponds to the in/out- modes, and

$$\omega_\lambda^\pm = \frac{1}{2}\left(\omega_\lambda^{\text{out}} \pm \omega_\lambda^{\text{in}}\right). \tag{14}$$

The corresponding eigenfunctions are related by the Bogoliubov transformation

$$g_\lambda^{(\text{in})} = \alpha_\lambda g_\lambda^{(\text{out})} + \beta_\lambda g_\lambda^{(\text{out})*}, \tag{15}$$

where α_λ and β_λ are the Bogoliubov coefficients. Using the linear relation between hypergeometic functions, similar to (Birrell and Davies, 1982) for the coefficients in this formula one finds

$$\alpha_\lambda = \left(\frac{\omega_\lambda^{\text{out}}}{\omega_\lambda^{\text{in}}}\right)^{1/2} \frac{\Gamma\left(1 - \imath\omega_\lambda^{\text{in}}\eta_0\right)\Gamma\left(-\imath\omega_\lambda^{\text{out}}\eta_0\right)}{\Gamma(-\imath\omega^+\eta_0)\lambda\Gamma(1 - \imath\omega_\lambda^+\eta_0))}, \tag{16}$$

$$\beta_\lambda = \left(\frac{\omega_\lambda^{\text{out}}}{\omega_\lambda^{\text{in}}}\right)^{1/2} \frac{\Gamma\left(1 - \imath\omega_\lambda^{\text{in}}\eta_0\right)\Gamma(\imath\omega_\lambda^{\text{out}}\eta_0)}{\Gamma(\imath\omega_\lambda^-\eta_0)\Gamma(1 + \imath\omega_\lambda^-\eta_0)}. \tag{17}$$

The mean number of particles produced through the modulation of the single scalar mode is

$$\langle \text{in}|N_\lambda|\text{in}\rangle = |\beta_\lambda|^2 = \frac{\sinh^2(\pi\omega_\lambda^-\eta_0)}{\sinh\left(\pi\omega_\lambda^{\text{in}}\eta_0\right)\sinh\left(\pi\omega_\lambda^{\text{out}}\eta_0\right)}. \tag{18}$$

The total number of particles produced is obtained by taking the sum over all the oscillation modes:

$$\langle \text{in}|N|\text{in}\rangle = \sum_{\lambda=1}^{\infty} \frac{\sinh^2[\pi\eta_0(\sqrt{\lambda^2 + (A+B)m^2} - \sqrt{\lambda^2 + (A-B)m^2})/2]}{\sinh(\pi\eta_0\sqrt{\lambda^2 + (A+B)m^2})\sinh(\pi\eta_0\sqrt{\lambda^2 + (A-B)m^2})}. \tag{19}$$

Therefore the energy related to the particles production is given by

$$E = \sum_{\lambda=1}^{\infty} N_\lambda \omega_\lambda^{\text{out}} \tag{20}$$

$$= \sum_{\lambda=1}^{\infty} \frac{\sinh^2[\pi\eta_0(\sqrt{\lambda^2+(A+B)m^2}-\sqrt{\lambda^2+(A-B)m^2})/2]}{\sinh(\pi\eta_0\sqrt{\lambda^2+(A+B)m^2})\sinh(\pi\eta_0\sqrt{\lambda^2+(A-B)m^2})}$$
$$\times\sqrt{\lambda^2+m^2(A+B)}.$$

3. CONCLUSION

The creation of particles from the vacuum takes place due to the interaction with dynamical external constraints. For example the motion of a single reflecting boundary (mirror) can create particles (Birrell and Davies, 1982), the creation of particles by time-dependent external gravitational field is another example of dynamical external constraints.

It has been shown (Nugayev and Bashkov, 1979; Nugayev, 1982) that particle creation by black hole in four dimension is as a consequence of the Casimir effect for spherical shell. It has been shown that the only existence of the horizon and of the barrier in the effective potential is sufficient to compel the black hole to emit black-body radiation with temperature that exactly coincides with the standard result for Hawking radiation. In this paper we have considered the particle creation in the spatially closed Robertson–Walker space-time. We considered a real massive scalar field which conformally coupled to the Robertson–Walker background. With the dependence of the scale factor on time, the case under consideration is a dynamical Casimir effect. When scale factor represent an asymptotically static space-time at past and future, the in- and out- vacuum states can be defined. Then we obtained the Bogoliubov coefficients, after that the number of particles produced and the energy related to those can be explicitly found.

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